Verifying the Equivalence of Representations of Joint Moment Vectors

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Abstract

Evaluating and verifying the moment data produced by clinical motion capture software can often be challenging. In particular, the software can generate several distinct component representations for the joint moment. These representations include the use of an orthonormal proximal frame, an orthonormal distal frame, and the non-orthogonal joint coordinate system. In this technical brief, a clear method is presented that can be used to demonstrate the equivalence of various representations. The method is illuminated using clinical data from a drop-vertical jump test, accommodates the difficulty that the frames used to express the moment vector using the joint coordinate system are non-orthogonal, and is applicable to representations of joint forces.

Keywords: Joint coordinate system, Joint rotations, Joint moments, Joint forces, Dual Euler basis

1. Introduction

The use of a joint coordinate system to describe the kinematics and kinetics associated with a biomechanical joint can be traced to the seminal paper by Grood and Suntay (1983) on the knee joint. Their work prompted a series of related investigations and standardizations for other anatomical joints (see Baker (2003) and Wu et al. (2005)). In the joint coordinate system, the first axis is attached to the proximal body, the third axis is attached to the distal body, and a second axis, known as a floating axis, is chosen to be orthogonal to the two other axes. If a set of Euler angles is used to describe the rotation of the joint, then each one of the three axes can be placed
in one-to-one correspondence with the axes used to define the Euler angles. The latter set of axes are known as the Euler basis.

Suppose the rotation of the joint is characterized by a rotation $R$ that serves to relate a set of right-handed orthonormal basis vectors, which we denote by $\{p_1, p_2, p_3\}$, attached to the proximal body to a set of right-handed orthonormal basis vectors, which we denote by $\{d_1, d_2, d_3\}$, attached to the distal body (cf. Figure 1). As discussed in the aforementioned papers, the choice of these sets are defined by a standardized set of anatomical landmarks. It is well known in the biomechanics literature (see (Desroches et al., 2010; Dumas and Chèze, 2014; O’Reilly et al., 2013) and references therein), that the Euler basis, which we denote by $\{g_1, g_2, g_3\}$, is not orthogonal. In addition, the Euler basis has a companion basis, known as the dual Euler basis, $\{g^1, g^2, g^3\}$ which features in representations of the moment vector in a joint coordinate system (O’Reilly, 2007; O’Reilly et al., 2013).

The joint moment vector $M$ has a variety of equivalent representations and in seeking clinical interpretations for $M$ one is often led to explore these representations (see, e.g., Schache and Baker (2007)). What is often not apparent is how to check that the representations are equivalent in particular when the joint coordinate system is being used. This situation is particularly the case when commercial or open-source programs are being used where the user has limited access to the algorithms producing the representations. Here, we present a straightforward method to check the equivalence. While the experimental data we use is from a drop-vertical jump test and pertains to the knee joint, our methods are presented in such generality that they can be readily applied to other anatomical joints and motion tasks.

To make this work as widely accessible as possible, a detailed discussion of the transformations between the various set of basis vectors and components can be found in the supplementary electronic resources that accompany this Technical Brief.

2. Methods

2.1. Acquisition of Sets of Kinematic and Force Data.

Participant movement data was used from a larger motion analysis study to monitor return-to-play criteria after anterior cruciate ligament injury. The study was approved by University
of California, San Francisco, Committee on Human Research (IRB# 12-10253). Retro-reflective spherical markers, 14 mm in diameter, were positioned on the participant on key bony landmarks and laterally on limb segments in rigid clusters. Three-dimensional kinematic data were collected at 250 Hz by a 10-camera system (Vicon, Oxford Metrics, UK). Kinetic data were collected at 1000 Hz by two embedded force platforms (AMTI, Watertown, MA USA).

In order to define the reference posture, a static trial was first conducted. Next, data was collected from a participant performing a drop-vertical jump as described by Hewett et al. (2005). The stance/impact phase was defined as the participant’s initial contact with the force platform following the drop, to the take-off for the maximum vertical jump. Kinematics for six-degree-of-freedom models of the thigh, shank, and foot along with force data from one of the force platforms were imported into Visual3D (C-Motion, Germantown, MD, USA). Marker position data was filtered using a 4th-order Butterworth filter with a cutoff frequency of 15 Hz. For this phase of testing, variables, including right knee joint angles and internal knee moment components in the proximal, distal, and joint coordinate system (see (4)1,2,4 below), were exported from Visual3D to MATLAB (MathWorks, Natick, MA, USA). The moment in question is an estimate of the moment $\mathbf{M}$ at the proximal end of the shank (cf. (Robertson et al., 2013, pp.161–164)). The latter program, in conjunction with the methods discussed below, was used to check the equivalence of representations of the moment in different coordinate systems.

2.2. Representations of Moments

For the purposes of illustration, we follow Grood and Suntay (1983) and assume that a 1-2-3 set of Euler angles ($1 = \alpha$, $2 = \beta$, $3 = \gamma$) is being used to parameterize the rotation of the knee joint. For this set of angles, $\alpha$ denotes the flexion-extension angle, $\beta$ denotes the adduction-abduction angle, and $\gamma$ denote the internal-external rotation. In addition, we have the following Euler and dual Euler basis sets, respectively:\footnote{For ease of exposition, we tacitly assume that $\beta \neq \pm \frac{\pi}{2}$.}

$$
\mathbf{g}_1 = \mathbf{p}_1, \quad \mathbf{g}_2 = \frac{\mathbf{d}_3 \times \mathbf{p}_1}{\|\mathbf{d}_3 \times \mathbf{p}_1\|}, \quad \mathbf{g}_3 = \mathbf{d}_3, \quad (1)
$$
and
\[ g^1 = \sec^2(\beta) g_1 - \sec(\beta) \tan(\beta) g_3, \quad g^2 = g_2, \quad g^3 = -\sec(\beta) \tan(\beta) g_1 + \sec^2(\beta) g_3, \quad (2) \]

It should be noted that \( g^1 \) and \( g^3 \) have no apparent clinical interpretations unlike \( g_1 \) and \( g_3 \) which are specified by bony landmarks (cf. Figure 1).

The moment vector \( \mathbf{M} \) has a variety of representations:

\[
\begin{align*}
\mathbf{M} &= M_1 \mathbf{p}_1 + M_2 \mathbf{p}_2 + M_3 \mathbf{p}_3 & \text{Proximal basis} \\
&= m_1 \mathbf{d}_1 + m_2 \mathbf{d}_2 + m_3 \mathbf{d}_3 & \text{Distal basis} \\
&= M_E^1 g_1 + M_E^2 g_2 + M_E^3 g_3 & \text{Euler basis} \\
&= M_{E_1} g^1 + M_{E_2} g^2 + M_{E_3} g^3. & \text{Dual Euler basis} \quad (3)
\end{align*}
\]

Explicit relationships between the four sets of basis vectors appearing in these representations can be found in the supplemental material accompanying this technical brief. The interested reader is also referred to (O’Reilly et al., 2013, Fig. 1(c)) for a graphical representation of (3)_{3,4}. The four sets of components in (3) are determined by projecting \( \mathbf{M} \) onto a specific basis vector:

\[
M_k = \mathbf{M} \cdot \mathbf{p}_k, \quad m_k = \mathbf{M} \cdot \mathbf{d}_k, \quad M_E^k = \mathbf{M} \cdot g^k, \quad M_{E_k} = \mathbf{M} \cdot g_k, \quad (k = 1, 2, 3). \quad (4)
\]

As we shall see below, the moment components in a joint coordinate system are typically reported in the literature are \( M_{E_1}, M_{E_2} \) and \( M_{E_3} \). That is,

\[
\begin{align*}
M_{E_1} &= \mathbf{M} \cdot g_1 = \mathbf{M} \cdot \mathbf{p}_1 = M_1, \\
M_{E_2} &= \mathbf{M} \cdot g_2 = \mathbf{M} \cdot \left( \frac{\mathbf{d}_3 \times \mathbf{p}_1}{||\mathbf{d}_3 \times \mathbf{p}_1||} \right) = \mathbf{M} \cdot g^2 = M_E^2, \\
M_{E_3} &= \mathbf{M} \cdot g_3 = \mathbf{M} \cdot \mathbf{d}_3 = m_3. \quad (5)
\end{align*}
\]

However, in order to show that the resulting vector is equivalent to its counterpart expressed in the proximal and distal bases, we need to compute \( M_{E_1} g^1 + M_{E_2} g^2 + M_{E_3} g^3 \). That is, knowledge of the Euler basis is sufficient to compute the components \( M_{E_k} \) of \( \mathbf{M} \) in the joint coordinate system, but, in order to construct the moment vector from these components, the dual Euler basis is required.

It is straightforward to show that \( \mathbf{M} \neq M_{E_1} g_1 + M_{E_2} g_2 + M_{E_3} g_3 \). Indeed, using the identity (S.16) from the supplemental material accompanying this brief, we find that

\[
M_{E_1} = M_E^1 + \sin(\beta) M_E^3, \quad M_{E_3} = \sin(\beta) M_E^1 + M_E^3. \quad (6)
\]
Thus, the difference in the components can be entirely attributed to the second Euler angle $\beta$. As mentioned in O’Reilly et al. (2013) for the knee joint this angle is small. For other joints, such as the shoulder, this is not the case but this angle still suffers from the “crosstalk” effect (cf. Dumas and Chèze (2014)). This effect can lead to challenges in comparing representations of the moment vector $\mathbf{M}$.

Concomitant with the representations for $\mathbf{M}$, related expressions for the magnitude $|\mathbf{M}|$ of $\mathbf{M}$ can be found by taking the inner (dot) product of this vector with itself and then taking the square root. After some rearranging, the resulting representations for $|\mathbf{M}|^2$ are

$$
|\mathbf{M}|^2 = \mathbf{M} \cdot \mathbf{M} = M_1^2 + M_2^2 + M_3^2 = m_1^2 + m_2^2 + m_3^2 = (M_E^1)^2 + (M_E^2)^2 + (M_E^3)^2 + 2 (M_E^1 M_E^3) \sin (\beta) = (M_{E1})^2 + (M_{E2})^2 + (M_{E3})^2 + ((M_{E1})^2 + (M_{E3})^2) \tan^2 (\beta) - 2 (M_{E1} M_{E3}) \sec (\beta) \tan (\beta).
$$

(7)

The underbraced terms in these representations arise because of the non-orthogonality of the Euler and dual Euler basis vectors.2

2.3. Signal Comparisons

In order to assess the equivalency of representations, software reported moment components $M_k$ and $m_k$ were used to provide estimates of $M_{Ek}$. The estimates for the components $M_{Ek}$ obtained using $M_k$ are denoted by $A_{Ek}$ while the estimates for the components $M_{Ek}$ obtained using $m_k$ are denoted by $b_{Ek}$. The appropriate transformations are obtained using (S.15) in the supplemental

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2The underbraced terms in (7) have been previously noted in the context of optimal control strategies which seek to optimize the magnitude of an applied torque. The interested reader is referred to Bloch et al. (2009) and O’Reilly (2014) for further details.
data:

\[
\begin{bmatrix}
A_{E_1} \\
A_{E_2} \\
A_{E_3}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\alpha) & \sin(\alpha) \\
\sin(\beta) & -\sin(\alpha)\cos(\beta) & \cos(\alpha)\cos(\beta)
\end{bmatrix}
\begin{bmatrix}
M_1 \\
M_2 \\
M_3
\end{bmatrix},
\]

\[
\begin{bmatrix}
b_{E_1} \\
b_{E_2} \\
b_{E_3}
\end{bmatrix} =
\begin{bmatrix}
\cos(\beta)\cos(\gamma) & -\cos(\beta)\sin(\gamma) & \sin(\beta) \\
\sin(\gamma) & \cos(\gamma) & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
m_1 \\
m_2 \\
m_3
\end{bmatrix}.
\] (8)

An RMS (evaluated at 1% of stance intervals) was calculated for each of the three components \(A_{E_k}\) and \(b_{E_k}\) and compared to \(M_{E_k}\). In addition, the moment magnitude \(|M|\) was calculated using \(M_k\) and \(m_k\) using (7)_{2,3} and compared to corresponding expression computed using the components \(M_{E_k}\) and (7)_{5}.

3. Results

Knee joint angle components from one repetition of a drop-vertical jump are presented in the inset of Figure 1 and used in the calculations (7, 8) of the estimates \(|M|, A_{E_k},\) and \(b_{E_k}\). Software produced moment components \(M_k, m_k\) and \(M_{E_k}\) for the drop-vertical jump are shown in Figure 2(a,b,d). We find that the RMS differences between \(A_{E_k}\) and \(M_{E_k}\) are 0.0 Nm, 8.2 \times 10^{-1} \text{ Nm}, 3.1 \times 10^{-5} \text{ Nm}, and between \(b_{E_k}\) and \(M_{E_k}\) are 4.4 \times 10^{-6} \text{ Nm}, 8.2 \times 10^{-1} \text{ Nm}, 0.0 \text{ Nm}, for the 1st, 2nd, and 3rd components, respectively. Moment magnitude from the software produced moment components \(M_k, m_k\) and \(M_{E_k}\) are displayed in Figure 3(a). The RMS difference between the \(|M|\) computed using \(M_k\) and \(M_{E_k}\) is 1.9 \times 10^{-1} \text{ Nm}, using \(m_k\) and \(M_{E_k}\) is 1.9 \times 10^{-1} \text{ Nm}, and using \(M_k\) and \(m_k\) is 4.4 \times 10^{-6} \text{ Nm}. Moment vectors (3)_{1,2,4} at 0% to 100% of stance in increments of 10% were all rotated into the proximal coordinate system and plotted in Figure 3(b). At each increment, the moment vectors visually align.

4. Discussion

It is difficult to conclude if \(M_{E_2}\) is correct from the exported signal data (Figure 2(a,b,d)). However, using rotated \(M_k\) and \(m_k\) signals allows us to compare moment vector components. The
small RMS values for $M_{E_2}$ confirm the similarity of the signals. Both exported signal data and RMS values demonstrate $M_1$ aligns with $M_{E_1}$ and $m_3$ aligns with $M_{E_3}$, as expected since $p_1 = g_1$ and $d_3 = g_3$. Reconstructing these components in the proximal coordinate system in Figure 3(b) also aids in confirming the equivalence of these signals.

Further comparison of overall magnitude gives confidence that we have obtained complete signals. All magnitude comparisons yield small values of RMS difference. However in this example, erroneously omitting the underbraced terms in the determination of $\|M\|$ (7) increases the RMS error value by a factor of 22. This magnitude calculation is dependent on $\beta$ due to the additional terms introduced by the non-orthogonal dual Euler basis. In the case of the knee where $\beta$ values are small, RMS error is also small. However, other joints such as the shoulder could see higher values of $\beta$ thus making the understanding and inclusion of the dual Euler basis even more critical.

5. Conclusions

To properly verify signals from commercially available software and confidently report findings from experimental data, one must understand the relationship between the different coordinate systems used to present biomechanical data. Confirmation of the $M_{E_1}$ and $M_{E_3}$ signals proves to be relatively simple. Calculating $M_{E_2}$ and $\|M\|$ proves to be more challenging and an understanding of the Euler basis and dual Euler basis is required. Improper handling can result in erroneous reporting especially at large values of $\beta$. The authors encourage the inclusion and use of the dual Euler basis in biomechanics education curriculum so that a more comprehensive understanding of the kinetic quantities described using the joint coordinate system can be achieved.

6. Conflicts of Interest

None of the authors have a conflict of interest.

7. Acknowledgements

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Figure 1: Schematic of the right knee joint showing the proximal \( \{p_1, p_2, p_3\} \) and distal \( \{d_1, d_2, d_3\} \) bases which corotate with the femur and tibia, respectively. The Euler and dual Euler basis vectors associated with the rotation of this joint when it is parameterized using a set of 1-2-3 Euler angles (as in Grood and Suntay (1983)) and the condyles C and D are also shown. The inset image shows the variation in the right knee joint angles during a drop-vertical jump test.
Figure 2: The moment vector $\mathbf{M}$ and four different representations of its components as a function of the percentage of the stance. (a) The components in the proximal basis $M_k = \mathbf{M} \cdot \mathbf{p}_k$, (b) the components in the distal basis $m_k = \mathbf{M} \cdot \mathbf{d}_k$, (c) the components $M^E_k = \mathbf{M} \cdot \mathbf{g}^k$ computed using (S.16), and (d) the components in the joint coordinate system $M^E_k = \mathbf{M} \cdot \mathbf{g}_k$. As anticipated $M^E_1 = M_1$ and $M^E_3 = m_3$. The data shown in (a), (b), and (d) are imported directly from Visual3D.
Figure 3: (a) Computation of the magnitude of the moment vector using three representations \( \mathbf{M}_{2,3,5} \) (i.e., the components in proximal, distal, and joint coordinate system representations). (b) The moment vector \( \mathbf{M} \) obtained by reconstructing \( \mathbf{M} \) from its components using \( \mathbf{M}_{1,2,4} \). The axes in this figure are not to scale.